4.1

CHECK Your Understanding

	K	Black
R	RK	RBl
Blu	BK	BBl
G	GK	GBl

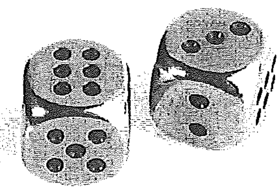
= 6 ✓

1. Cam is on vacation. In his suitcase he has three golf shirts (red, blue, and green) and two pairs of shorts (khaki and black).
   - **a)** Use an outcome table to count the total number of outfit variations he has for golfing. 6
   - **b)** Use the Fundamental Counting Principle to verify your result in part a). $3 \times 2 = 6$

2. Missy is buying her first new car. The model she wants comes in four colours (red, black, white, and silver) and she has a choice of leather or cloth upholstery.
   - **a)** Use a tree diagram to count all the upholstery–colour choices that are available.
   - **b)** Use the Fundamental Counting Principle to verify your result in part a). $4 \times 2 = 8$

red — leather, cloth
black — L, C
white — L, C
silver — L, C
= ? ✓

3. For each situation below, indicate whether the Fundamental Counting Principle applies and explain how you know.
   - **a)** Counting the number of possibilities when rolling a 3 or a 6 with a standard die no
   - **b)** Counting the number of outfit variations when selecting a shirt, a tie, and shoes to wear to the semiformal dance yes
   - **c)** Counting the number of possibilities when picking the winner in a stock car race in either the fourth, fifth, or sixth race of the evening no
   - **d)** Counting the number of possibilities to choose from when buying a car with either standard or automatic transmission, air conditioning or not, power windows or not, and GPS navigation or not yes $2 \times 2 \times 2 \times 2$



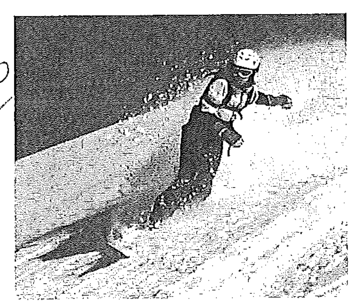
✓

PRACTISING

Game: 1 2 3
W — W ✓, L — W ✓, L
L — W — W ✓, L; L
✓

4. **a)** Kim plays hockey on the Lloydminster Ice Cats. Her team is in a best-two-out-of-three playoff series. Create a tree diagram to show all the win–loss possibilities for her team.
   b) Use your tree diagram to count the number of ways Kim's team can win the series despite losing one game. 2 ways

5. Xtreme clothing company makes snowboarding pants in five colours and sizes of small, medium, large, and extra large. How many different colour–size variations of snowboarding pants does this company make? $5 \times 4 = 20$ ✓



6. A computer store sells 5 different desktop computers, 4 different monitors, 6 different printers, and 3 different software packages. How many different computer systems can the employees build for their customers? $5 \times 4 \times 6 \times 3$
$= 360$ ✓

7. Jeb's Diner offers a lunch special. You have a choice of 3 soups, 5 sandwiches, 4 drinks, and 2 desserts. How many meals are possible if you choose one item from each category?

Lunch Special
Choose one soup, one sandwich, one drink, and one dessert.

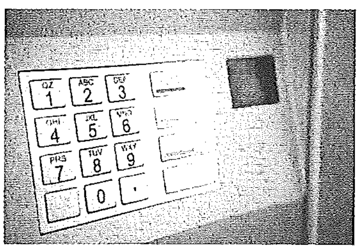
Soups	Sandwiches
soup of the day	pastrami, cheese
French onion	turkey, tuna
minestrone	grilled vegetables

Drinks
cola, root beer, milk, orange juice

Dessert
ice cream sundae
chocolate brownie

$3 \times 5 \times 4 \times 2$
$= 120$ ✓

8. Tom likes rap music and classic rock. His friend Charlene has 8 rap CDs, 10 classic rock CDs, and 5 country and western CDs in her car. How many CDs can Charlene select from to play in her car stereo that will match Tom's musical tastes? $10 + 8 = 18$ ✓



9. Rachelle's bank card has a five-digit PIN where each digit can be 1 to 9.
   a) How many PINs are possible if each digit can repeat? $9 \times 9 \times 9 \times 9 \times 9 = 59049$ ✓
   b) How many PINs are possible if each digit can be used only once?
   $9 \times 8 \times 7 \times 6 \times 5 = 15120$ ✓

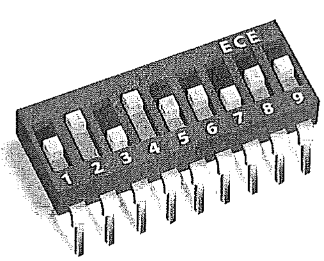
10. Computers code information in a binary sequence, using 0 or 1 for each term in the sequence. Each sequence of eight terms is called a byte (for example, 00110010). How many different bytes can be created?
   $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$ ✓

11. a) A country's postal code consists of six characters. The characters in the odd positions are upper-case letters, while the characters in the even positions are digits (0 to 9). How many postal codes are possible in this country? $26 \times 10 \times 26 \times 10 \times 26 \times 10 = 17576000$

LNL NLN

   b) Canadian postal codes are similar, except the letters D, F, I, O, and U can never appear. (This is because they might be mistaken for the letters E or V or the numbers 0 or 1.) How many postal codes are possible in Canada? $21 \times 10 \times 21 \times 10 \times 21 \times 10 = 9261000$ ✓

12. A small town in Manitoba has a phone area code of 204 and two different three-digit prefixes as shown: 204-945-□□□□ or 204-940-□□□□. How many different phone numbers are possible for this town? $2 \times 10 \times 10 \times 10 \times 10 = 20,000$ ✓



13. The code to a garage door opener is programmed by moving each of nine switches to any one of three positions. How many different codes are possible? $3^9 = 19683$ ✓

14. A vehicle rental company has 8 pickup trucks, 10 passenger vans, 35 cars, and 12 sports utility vehicles for rent. How many choices does a customer have when renting just 1 vehicle?

$8 + 10 + 35 + 12 = 65$ ✓

15. The "Pizza Shoppe" offers these choices for each pizza:

- thin or thick crust
- regular or whole-wheat crust
- 2 types of cheese
- 2 types of tomato sauce
- 20 different toppings

How can you determine the number of different pizzas that can be made as follows:

a) A pizza with any crust, cheese, tomato sauce, and 1 topping $2 \times 2 \times 2 \times 2 \times 20 = 320$

b) A pizza with a thin whole-wheat crust, tomato sauce, cheese, and no toppings $1 \times 1 \times 2 \times 2 = 4$

16. An Alberta licence plate has three letters followed by three digits; for example, ABC 123. The letters I and O are not used to avoid confusion with the digits 1 and 0.

a) How many different Alberta licence plates are possible? $24 \times 24 \times 24 \times 10 \times 10 \times 10 = 13\,824\,000$

b) Because of the growing number of vehicles, the province is changing to plates with three letters followed by four digits; for example, ABC 1234. How many more licence plates are possible? $24 \times 24 \times 24 \times 10 \times 10 \times 10 \times 10 =$

subtract b−a

$= 124\,416\,000$

Closing

17. Counting problems often involve several tasks that are described using the words AND and OR. What is the mathematical meaning behind these words, and how does this affect the strategy you would use to solve counting problems that involve these words? Support your answer using relevant examples.

Extending

18. a) Determine the likelihood that each of the following events can occur using a standard deck of cards.

- i) Drawing a king or a queen
- ii) Drawing a diamond or a club
- iii) Drawing an ace or a spade

b) Does the Fundamental Counting Principle apply to any situation in part a)? Explain.

19. How many two-digit numbers are not divisible by either 2 or 5?

20. A test has 10 true-false questions. A student attempts every question by guessing. What is the likelihood that the student will get a perfect score?

21. Recall Jeb's Diner and the lunch special from question 7. There are 3 soups, 5 sandwiches, 4 drinks, and 2 desserts to choose from. How many meals are possible if you do not have to choose an item from a category?

4.2

In Summary

Key Ideas

- A permutation is an arrangement of objects in a definite order, where each object appears only once in each arrangement. For example, the set of three objects *a*, *b*, and *c* can be listed in six different ordered arrangements or permutations:

	Position 1	Position 2	Position 3
Permutation 1	*a*	*b*	*c*
Permutation 2	*a*	*c*	*b*
Permutation 3	*b*	*a*	*c*
Permutation 4	*b*	*c*	*a*
Permutation 5	*c*	*a*	*b*
Permutation 6	*c*	*b*	*a*

- The expression $n!$ is called n factorial and represents the number of permutations of a set of n different objects and is calculated as

$$n! = n(n-1)(n-2)\ldots(3)(2)(1)$$

$7! = 7\cdot(7-1)(7-2)(7-3)\cdot$
$= 7\cdot 6\cdot 5\cdot 4\cdot 3$

Need to Know

- In the expression $n!$, the variable n is defined only for values that belong to the set of natural numbers; that is, $n \in \{1, 2, 3, \ldots\}$.

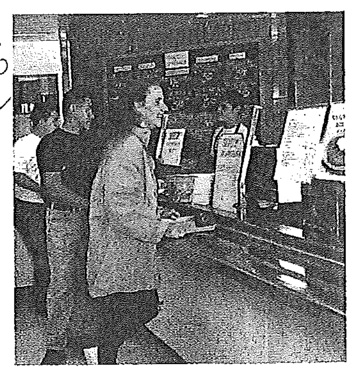
CHECK Your Understanding

1. Evaluate the following expressions.

   a) $6!$ $= 720$
   b) $9 \cdot 8!$ $= 9! = 362880$
   c) $\dfrac{5!}{3!}$ $= 20$
   d) $\dfrac{8!}{7!}$ 8
   e) $3! \cdot 2!$ $= 12$
   f) $\dfrac{9!}{4! \cdot 3!}$ $= 2520$

   ✓

2. a) How many permutations are possible of Ken, Sarah, and Raj when they line up to buy a slice of pizza? Describe your strategy. $3\cdot 2\cdot 1 = 6$
   b) Express the number of permutations using factorial notation. $3!$ ✓



3. Write the following expressions using factorial notation.

   a) $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ $= 5!$
   b) $9 \cdot 8 \cdot 7$ $\dfrac{9!}{6!}$
   c) $\dfrac{15 \cdot 14 \cdot 13}{4 \cdot 3 \cdot 2 \cdot 1}$ $\dfrac{15!}{12!\,4!}$
   d) $100 \cdot 99$ $\dfrac{100!}{98!}$

   ✓

4. Which expressions are undefined? Explain how you know.

   a) $(\cancel{-4})!$
   b) $7!$
   c) $\cancel{5.5}!$
   d) $\cancel{\dfrac{3}{4}}!$ ✓

   not natural. it's

PRACTISING

5. Evaluate the following expressions.

a) $8 \cdot 7 \cdot 6!$ $= 8! = 40320$

b) $\frac{12!}{10!}$ $= 132$

c) $\frac{8!}{2! \cdot 6!}$ $= \frac{8 \times 7}{2} = 28$

d) $\frac{7 \cdot 6!}{5!}$ $= \frac{7!}{5!} = 7 \cdot 6 = 42$

e) $4\left(\frac{6!}{2! \cdot 2!}\right)$ $= 720$

f) $4! + 3! + 2! + 1!$ $= 33$

6. Simplify each of the following expressions, where $n \in I$.

a) $\frac{n!}{(n-1)!}$ $= \frac{n(n-1)!}{(n-1)!} = n$

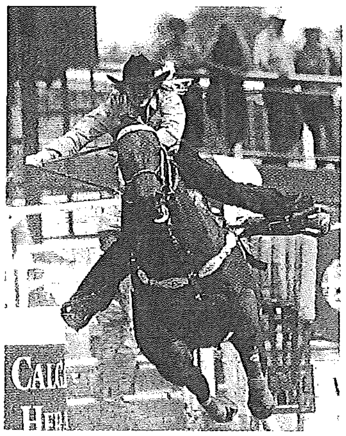
b) $(n+4)(n+3)(n+2)!$ $= (n+4)!$

c) $\frac{(n+1)!}{n!}$ $= n+1$

d) $\frac{n!}{(n-3)!}$ $= (n)(n-1)(n-2)$

e) $\frac{(n+5)!}{(n+3)!}$ $= (n+5)(n+4)$

f) $\frac{(n-2)!}{(n-1)!}$ $= \frac{1}{n-1}$



The Calgary Stampede, held every July, celebrates Western heritage. The first Stampede was held in 1912.

7. How many different permutations can be created when nine students line up to buy tickets for the afternoon rodeo at the Calgary Stampede?
$9! = 362\,880$

8. The environmental club has five members. They want to select a president, a vice-president, a secretary, a treasurer, and a spokesperson. How many different ways can this be done? $5! = 120$

9. Amy and Traci are planning a summer trip to Vancouver Island in British Columbia. They plan to spend six days in Tofino. While they are there, they have decided to take part in the following activities: whale watching, hiking, surfing, sea kayaking, snorkeling, and fishing. They plan to do a different activity each day. In how many different ways can they sequence these activities over the six days? $6!$

10. Visitors to a movie website will be asked to rank 28 movies. The website will present the movies in a different order for each visitor to reduce bias in the poll. How many permutations of the movie list are possible? $28! = 3.05 \times 10^{29}$

11. Solve for n, where $n \in I$.

a) $\frac{(n+1)!}{n!} = 10$ $= \frac{(n+1)n!}{n!}$ $n = 9$

b) $\frac{(n+2)!}{n!} = 6$

$\frac{(n+2)(n+1)n!}{n!} =$

$n^2 + 3n + 2 = 6$

$n^2 + 3n - 4 = 0$

$(n+4)(n-1) = 0$

$n = \cancel{-4}, 1$

can't be $n = -3$ since $(-3+2)! = (-1)!$ is impossible

c) $\frac{(n-1)!}{(n-2)!} = 8$ $= \frac{(n-1)(n-2)!}{(n-2)!}$ $n = 9$

d) $\frac{3(n+1)!}{(n-1)!} = 126$

12. A baseball coach is determining the batting order for the nine players he is fielding. The coach has already decided that he wants the pitcher to hit in last position. How many different batting orders are possible?

$8! = 40,320$

13. On an assembly line for a company that makes digital cameras, seven-digit serial numbers are assigned to each camera under the following conditions:
- Only the digits from 3 to 9 are used. = 7 choices
- Each digit is to be used only once in each serial number.

How many different serial numbers are possible? Express your answer using factorial notation and explain why your answer makes sense.

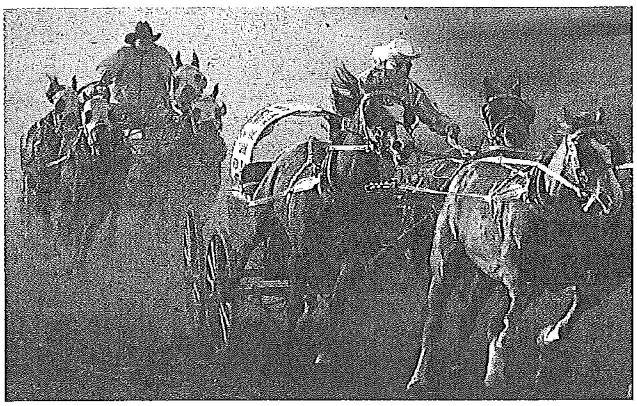
since no repetition
$\underset{\text{choices}}{7} \times \underset{\text{choices}}{6} \times 5 \times 4 \times 3 \times 2 \times 1$
$= 7!$ ✓

14. A model train has an engine, a caboose, a tank car, a flat car, a boxcar, a livestock car, and a refrigerator car. How many different ways can the cars be arranged between the engine and the caboose?

engine – $\underset{120}{5!}$ – caboose ✓

15. Eight drivers have made it to the final chuckwagon race at Back to Batoche Days, although Brant's wagon is considered certain to win. In how many different orders can the eight chuckwagons finish, if Brant's wagon wins?

$7! = 5040$ ways ✓



Each year for three days in July, Saskatchewan's Métis Nation gathers at the site of the Battle of Batoche for a festival called Back to Batoche Days. The Métis settled this area, northeast of Saskatoon, Saskatchewan, in the early 1870s.

Closing

16. Consider the word YUKON and all the ways you can arrange its letters using each letter only once.
    a) One possible permutation is KYNOU. Write three other possible permutations.
    b) Use factorial notation to represent the total number of permutations possible. Explain why your expression makes sense.

KNYOU
KYNUO
KNYUO ✓

$\underset{\text{choices}}{5} \times \underset{\text{choices}}{4} \times 3 \times 2 \times 1$
$= 5!$ each letter used only once.

Extending

17. a) For what values of n is $n!$ greater than 2^n?
    b) For what values of n is $n!$ less than 2^n?

18. Darlene and Arnold belong to the Asham Stompers, a 10-member dance troupe based in Winnipeg, Manitoba, that performs traditional Métis dances. During the Red River Jig, they always arrange themselves in a line, with Darlene and Arnold next to each other. How many different arrangements of the dancers are possible for the Red River Jig?

4.3

$_nP_r = \frac{n!}{(n-r)!}$

CHECK Your Understanding

1. Evaluate the following expressions.

a) $_5P_2$ $\frac{5!}{3!} = 20$ **c)** $_{10}P_5 = \frac{10!}{5!} = 30240$ **e)** $_7P_7$ $\frac{7!}{0!} = 7! = 5040$

b) $_8P_6$ $\frac{8!}{2!} = 20160$ **d)** $_9P_0$ $\frac{9!}{9!} = 1$ **f)** $_{15}P_5$ $\frac{15!}{10!} = 360360$

2. Katrina, Jess, Nazir, and Mohamad are running for student council.

a) Create a list or use a tree diagram to count all the ways that a president and vice-president can be elected.

K — J, N, M
J — K, N, M
N — K, J, M
M — K, J, N

b) Verify your results from part a) using the formula for $_nP_r$. $_4P_2 = \frac{4!}{(4-2)!} = 12$

3. How many different ways can six different chocolate bars be distributed to each set of children, if each child is to receive only one?

a) 4 children **b)** 1 child

a) $_6P_4 = \frac{6!}{(6-4)!} = 360$ b) $_6P_1 = \frac{6!}{(6-1)!} = 6$

4. Without calculating, predict which value is larger: $_{10}P_8$ or $_{10}P_2$. Explain your prediction.

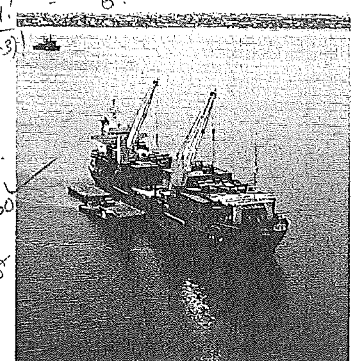
$\frac{10!}{(10-8)!}$ $\frac{10!}{(10-2)!}$

more ways to arrange 8 of 10 than 2 of 10

PRACTISING

5. Nine girls are available to fill three positions on a hockey team: centre, winger, and defence. If all the girls are equally competent in each position, how many different ways can the positions be filled?

$_9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = 504$



6. The photo club must select an executive committee to fill the roles of president, vice-president, treasurer, and secretary. All 15 members of the club are eligible. How many different executive committees are possible?

$_{15}P_4 = \frac{15!}{11!} = 32760$

7. The MV UMIAVUT (umiavut means "our boat" in Inuktitut) is a cargo ship that sails the Northwest Passage. If it carries eight different coloured signal flags, how many different signals could be created by hoisting all eight flags on the ship's flagpole in different orders?

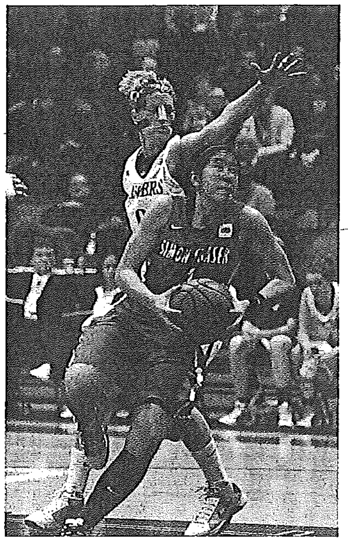
$_8P_8$ $\frac{8!}{0!} = 40320$

8. A wheelchair basketball team sold 5000 tickets in a draw to raise money. How many ways can the tickets be drawn to award the first, second, and third prizes if each winning ticket drawn is not put back prior to the next draw?

$5000 \cdot 4999 \cdot 4998$
$= 1.25 \times 10^{11}$
$_{5000}P_3 = \frac{5000!}{4997!}$

9. If a Canadian social insurance number begins with the digit 6, it indicates that the number was registered in Manitoba, Saskatchewan, Alberta, Northwest Territories, or Nunavut. If the number begins with a 7, the number was registered in British Columbia or the Yukon. How many different SINs can be registered in each of these groups of provinces and territories?

$1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
$= 100\,000\,000$



10. Sandy belongs to the varsity basketball team. There are 12 students on the team. How many ways can the coach select each of the following?

$_{12}P_5 = \frac{12!}{7!} = 95040$

a) The starting five players (a point guard, a shooting guard, a small forward, a power forward, and a centre)

$_{11}P_4 = \frac{11!}{7!} = 7920$

b) The starting five players, if the tallest student must start at the centre position

$_{10}P_3 = \frac{10!}{7!} \times 2 = 1440$

c) The starting five players, if Sandy and Natasha must play the two guard positions

can switch

11. For each expression, state the values of n for which the expression is defined.

a) $\dfrac{n!}{(n-1)!}$ $n \geq 1$

b) $(n + 4)(n + 3)(n + 2)!$ $n \geq -2$

c) $\dfrac{(n + 1)!}{n!}$ $n \geq 0$

d) $\dfrac{(n + 5)!}{(n + 3)!}$ $n \geq -3$

$n+2=0$
$n=-2$

12. There are six different marbles in a bag. Suppose you reach in and draw one at a time, and do this four times. How many ways can you draw the four marbles under each of the following conditions?

a) You do not replace the marble each time.

b) You replace the marble each time.

c) Compare your answers for parts a) and b). Does it make sense that they differ? Explain.

13. How many ways can five different graduation scholarships be awarded to 20 students under each of these conditions?

a) No student may receive more than one scholarship.

b) There is no limit to the number of scholarships awarded to each student.

14. All phone numbers consist of a three-digit area code, a three-digit exchange, and then a four-digit number. A town uses the 587 area code and the exchange 355; for example, 587-355-1234. How many different phone numbers are possible in this town under the following conditions?

a) There are four different digits in the last four digits of the phone number.

b) At least one digit repeats in the last four digits of the phone number.

$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$

$n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$n = 5,\ \cancel{n = -4}$

$\frac{n!}{(n-2)!} = 20 = 5 \times 4$

$= \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}}$

15. Solve each equation for n. State any restrictions on n.

a) $_nP_2 = 20$ **b)** $_{n+1}P_2 = 72$

16. Solve each equation for r. State any restrictions on r.

a) $_6P_r = 30$ **b)** $2(_7P_r) = 420$

a) $_6P_r = 30$

$= \frac{6!}{(6-r)!} = 30 = 6 \cdot 5$

$= \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6 \cdot 5$

17. Show that for all values of n, where $n \in \mathrm{N}$,

$$_nP_n = {_nP_{n-1}}$$

Ch4 Mid-Ch Review

PRACTISING

Lesson 4.1

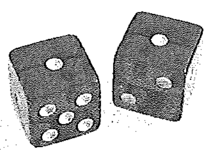
1. A sub shop offers the following choices:
   - 3 types of buns
   - 5 types of cold cuts
   - 3 types of cheese
   - 12 different toppings
   - 3 different sauces

   If Mario wants a sub with one item from each category, how many different subs can he choose from? $3 \cdot 5 \cdot 3 \cdot 12 \cdot 3 = 1620$ ✓

2. Radio and television stations in the USA and Canada have four-character station names. There are 3 choices for the first letter: K, W, and C, and a blank may be used as the last character. For instance, both CKLW and WHO are acceptable names. How many station names are possible? ✓
   $3 \times 26 \times 26 \times 27 = 54756$

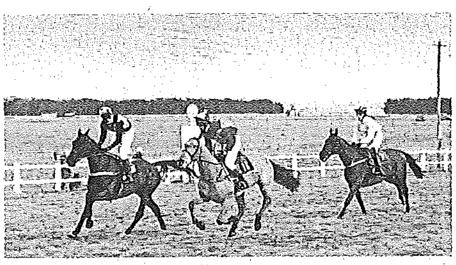
3. How many ways can a sum of 2 or a sum of 10 be rolled with a pair of dice? (or = add)

   1,1
   4,6 ; 6,4
   5,5
   So 4 ways ✓



4. How many ways can you select 3 horses to come first, second, and third in a 10-horse race?

   $10 \cdot 9 \cdot 8 = 720$ ✓



Lesson 4.2

5. Evaluate each expression.

   a) $8!$ 40320 b) $6! \cdot 3!$ 4320 c) $\dfrac{9!}{6!} = 504$ d) $\dfrac{10 \cdot 9!}{5 \cdot 8!} = 18$

6. How many different lineups can be formed by nine players on a softball team?

   $9! = 362\,880$

7. Simplify each expression.

   a) $(n+5)(n+4)! = (n+5)!$
   b) $\dfrac{(n+4)!}{(n+2)!} = (n+4)(n+3)$
   c) $\dfrac{(n-4)!}{(n-5)!} = (n-4)$ ✓
   d) $\dfrac{(n+2)!}{n!} = (n+2)(n+1)$

8. Solve each equation.

   a) $\dfrac{n!}{(n-2)!} = 72$ ✓ ($n=9$, not -8)
   $n(n-1) = 72$
   $n^2 - n - 72 = 0$
   $(n-9)(n+8) = 0$

   b) $\dfrac{(n-1)!}{(n-3)!} = 30$ ($n=7$, not -4) ✓
   $(n-1)(n-2) = 30$
   $n^2 - 3n + 2 = 30$
   $n^2 - 3n - 28 = 0$
   $(n-7)(n+4) = 0$

Lesson 4.3

9. Evaluate the following expressions.

   a) ${}_9P_2 = \dfrac{9!}{7!} = 72$ b) ${}_{12}P_8 = \dfrac{12!}{4!} = 19{,}958{,}400$ c) ${}_5P_5 = 5! = 120$ d) ${}_{12}P_{10} = \dfrac{12!}{2!} = 2.395 \times 10$

10. State the restrictions on the variable, n, for each expression in ✓
    a) question 7 a) $n \geq -4$ b) $n \geq -2$ c) $n \geq 5$ d) $n \geq 0$
    b) question 8 a) $n \geq 2$ b) $n \geq 3$

11. Rennie has 20 CDs in his car. His CD player holds 6 CDs. How many different ways can he load his CD player? ${}_{20}P_6 = \dfrac{20!}{14!} = 27{,}907{,}200$ ✓

12. Manny is the captain of the 15-member soccer team that has won his city's championship. How many ways can Manny and 2 (any) other players line up to receive the championship trophy, if the captain must be first in line? ✓ $1 \times 14 \times 13 = 182$

13. Margo claims that for some counting problems, you can use either the Fundamental Counting Principle or the permutations formula,

$${}_nP_r = \frac{n!}{(n-r)!}$$

Do you agree? Use examples to support your position. yes ✓

dial # Lock that can't have repeating #'s
fund count: $10 \cdot 9 \cdot 8 \cdot 7 =$ same
perm.: $\dfrac{10!}{(10-4)!} =$